

# A SIMPLIFIED 2D GRADED MESH FD-TD ALGORITHM FOR CALCULATING THE CHARACTERISTIC IMPEDANCE OF SHIELDED OR OPEN PLANAR WAVEGUIDES WITH FINITE METALLIZATION THICKNESS

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## ABSTRACT

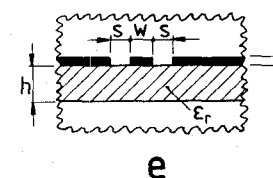
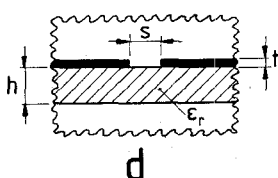
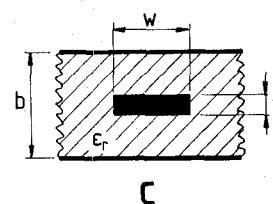
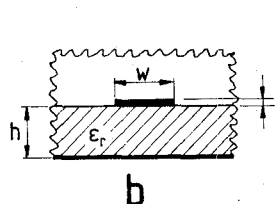
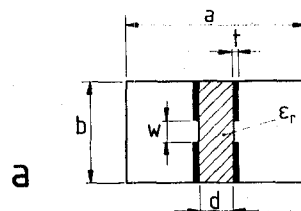
A full-wave finite-difference time-domain (FD-TD) algorithm is described for the efficient calculation of the characteristic impedance of planar waveguiding structures including the finite metallization thickness. The FD-TD algorithm is based on a two-dimensional graded mesh combined with adequately formulated absorbing boundary conditions. This allows the inclusion of nearly arbitrarily shaped, fully or partially lateral open or shielded guiding structures with layers of finite metallization thickness. Moreover, by a modified formulation, an actual 2D grid for 2D problems is obtained, i.e. the grid size for these problems is zero in  $z$ -direction as long as the waveguide is homogeneous in that direction. The characteristic impedances are calculated by using the related adequate power-voltage or power-current definitions, respectively, for structures suitable for usual integrated circuits, such as bilateral finlines, open microstrip line, lateral open triplate line, open slot-line, and open coplanar line. The theory is verified by comparison with results obtained by other methods.

## INTRODUCTION

Shielded, open or partially open waveguide structures of the class shown in Fig. 1 have found increasing interest for integrated circuit applications in the millimeter-wave frequency range [1] – [13]. As this class includes a wide variety of specially shaped waveguiding structures used, in the design of integrated circuits, it is highly desirable to dispose of a reliable computer analysis which is sufficiently general and flexible to allow accurate calculations of the characteristic impedance of all desired cases including open or partially open structures.

Various methods of analyzing one or several of the structures of Fig. 1 have been the subject of many papers, including especially the spectral-domain method [8] – [10]. Although, in principle, the finite metallization thickness can be considered by the spectral-domain approach as well, as for the criteria of flexibility, accuracy and computational efficiency, the FD-TD method is considered to be a very appropriate candidate for calculating the characteristic impedance for transmission lines with particular cross-sectional shapes, such as those shown in Figs. 1 a – e. The conventional FD-TD approach, however, utilizing the three-dimensional Yee's mesh for appropriate resonating sections requires considerable cpu time, needs a relatively large memory size, and tends to inaccuracies in the near of the cutoff frequencies.

A new two-dimensional FD-TD formulation has been introduced very recently [7], [11] which helps to alleviate the above mentioned shortcomings of the conventional FD-TD approach. The purpose of this paper is to extend the two-dimensional FD-TD method introduced in [7] by involving an actual 2D grid formulation, adequate absorbing boundary conditions, a graded mesh algorithm, and an efficient calculation procedure for the characteristic impedance. The grid size for these FD-TD problems is zero in  $z$ -direction as long as the waveguide is homogeneous in that direction. Numerical results will be presented to elucidate the usefulness of the method. The theory is verified by comparison with results available from other methods.



**Fig. 1:**

Investigated class of shielded, partially or fully open millimeter-wave waveguide structures with finite metallization thickness

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## THEORY

The usual FD-TD method is formulated by discretizing Maxwell's curl equations over a finite volume and approximating the derivatives with centered difference approximations [1]. The finite difference field relations in the direction  $z$  of the wave propagation, however, may be advantageously formulated by using the complex notation of [7], [11]. This leads to a 2D FD-TD formulation which reduces significantly the cpu time and storage requirements.

A further reduction is obtained when, in contrast to [7], [11], the phase factors  $\beta$  are directly introduced, rather than using a uniform mesh extension of  $\Delta l$  in  $z$ -direction. This yields

$$\vec{E}(z) = \vec{E}(0) e^{-j\beta z}; \quad \frac{\partial \vec{E}^n}{\partial z} = -j\beta \vec{E}^n(z). \quad (1)$$

The same is true for the H field.

Implementing (1) in the FD-TD Yee's formulation, a reduced set of 2D FD-TD equations is obtained for all six field components. The result is shown for  $H_x$  as an example,

$$H_x^{n+1/2}(i,j+1/2) = H_x^{n-1/2}(i,j+1/2) + s/[\mu_{xx}(i,j+1/2) \cdot Z_{F0}] \cdot \left\{ -j\beta \Delta l E_y^n(i,j+1/2) + \frac{1}{q} [E_z^n(i,j) - E_z^n(i,j+1)] \right\}, \quad (2)$$

where the stability factor is  $s = c \Delta t / \Delta l$ ,  $c$  is the velocity of light,  $Z_{F0}$  is the characteristic impedance of free space, and  $\mu_{xx}$  is the diagonal element of the relative permeability tensor. The graded mesh parameters are  $\Delta x = p \Delta l$ ,  $\Delta y = q \Delta l$  with the graded mesh scaling factors  $p$ ,  $q$ . The remaining finite difference equations related to the other five field components can be similarly calculated.

The differentiation in propagation direction is performed analytically (1). This leads advantageously to a reduced mesh (Fig. 2) and higher accuracy as compared with the recent 2D FD-TD formulation [7], [11]. The absorbing boundary conditions for the 2D FD-TD method presented are formulated according to Mur [5].

The principal numerical calculation steps in the two-dimensional FD-TD algorithm are similar to those in the conventional FD-TD approach with the exception that a propagation factor  $\beta$  has to be selected first. After placing the boundary conditions, launching an excitation pulse, waiting until the distribution of the pulse is stable and performing the Fourier transformation, the modal frequencies related to the selected propagation factor are obtained.

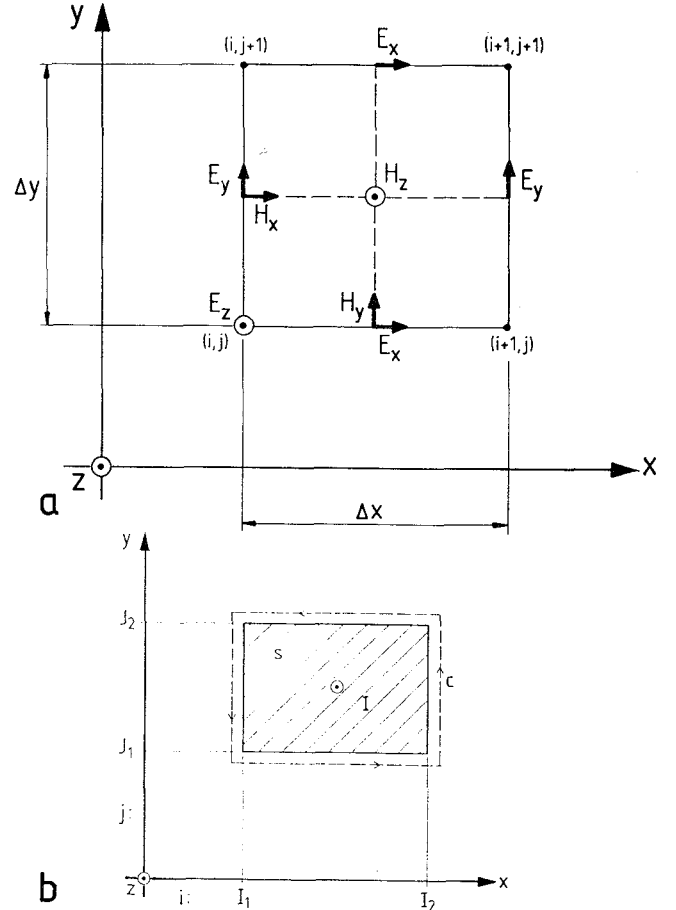
For calculating the characteristic impedance, the related field components have to be determined by an adequate procedure. The DFT algorithm is applied to the  $E, H(i,j)$  matrix for which the calculation is continued for a few periods at the resonance frequency. The required values for an adequate characteristic impedance definition, power  $P$ , voltage  $V$ , current  $I$ , respectively, are found by numerical integration (cf. also Figs. 2a,b):

$$P = \frac{1}{2} \operatorname{Re} \left[ \int_s (\vec{E} \times \vec{H}^*) d\vec{s} \right] \cong \frac{1}{2} \Delta l^2 \operatorname{Re} \left[ \sum_i \sum_j (E_x H_y^* - E_y H_x^*)_{i,j} p_i q_j \right] \quad (3)$$

$$V = \int_1^2 \vec{E} d\vec{l}, \quad (4)$$

$$I = \oint \vec{H} d\vec{l} - \int_s \epsilon \frac{\partial \vec{E}}{\partial t} d\vec{s} \cong \Delta l \left[ \sum_{i1}^{I2} p_i (\Pi_x(i, J1-1/2) - H_x(i, J2+1/2)) - \sum_{j1}^{J2} q_j (H_y(I1-1/2, j) - H_y(I2+1/2, j)) \right] - j\omega \epsilon_0 \Delta l^2 \sum_{i1}^{I2} \sum_{j1}^{J2} p_i q_j \epsilon_{rz} E_z(i, j) \quad (5)$$

For calculations of the characteristic impedance for more frequency points, a significant reduction in cpu time can be achieved, if the field distribution of the last iteration step is utilized for the excitation field for the next step. This is due to the fact that only a reduced number of iterations is then required for obtaining a steady state.

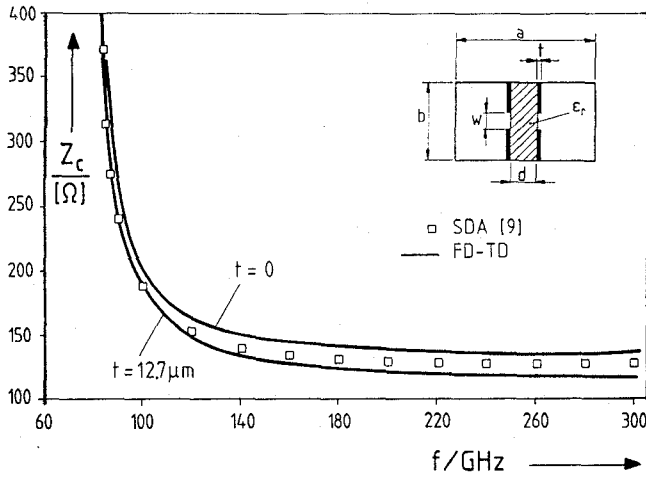


**Fig. 2:**  
Reduced two-dimensional FD-TD algorithm  
a) Pure two-dimensional mesh (i, j)  
b) Calculation path for the current

## RESULTS

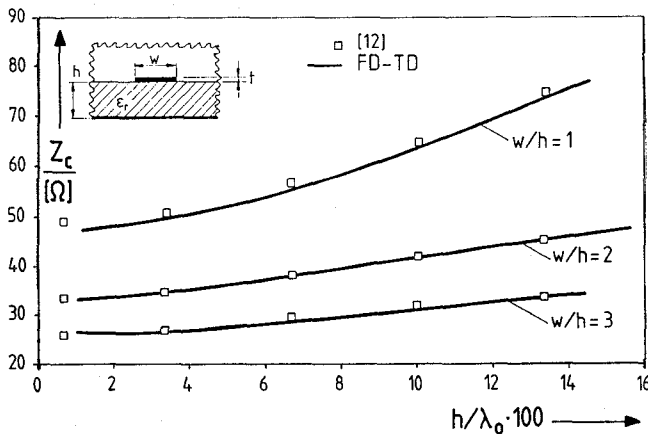
Fig. 3 presents the characteristic impedance (power-voltage definition) for a bilateral finline on quartz substrate for two metallization thicknesses  $t = 0$ , and  $t = 12.7\mu\text{m}$ . The results are in good agreement with the spectral-domain approach (SDA) [9] for  $t = 0$  although a very poor discretization ( $20 \times 20$ ) with a graded mesh factor of  $p > 10$  (in  $x$ -direction) has been used.

The power-current characteristic impedance definition is applied for the open microstrip line (Fig. 4). The values for three different  $w/h$  ratios compare well with those reported in [12]. Fig. 5 presents the related values for a lateral open triplate structure in comparison with results from analytical formulas given in [13].



**Fig. 3:**

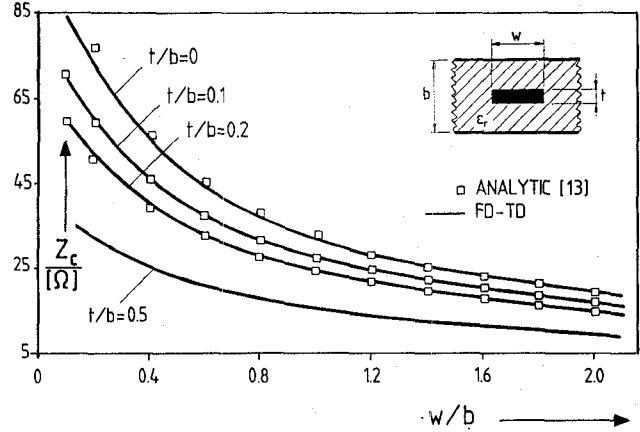
Characteristic impedance (power-voltage definition) for a bilateral finline on quartz substrate for two metallization thicknesses  $t = 0$ , and  $t = 12.7\mu\text{m}$ . WR-3 housing,  $w = 0.1\text{mm}$ ,  $d = 0.127\text{mm}$ . Discretization  $20 \times 20$ , graded mesh factor  $p > 10$  (in  $x$ -direction).



**Fig. 4:**

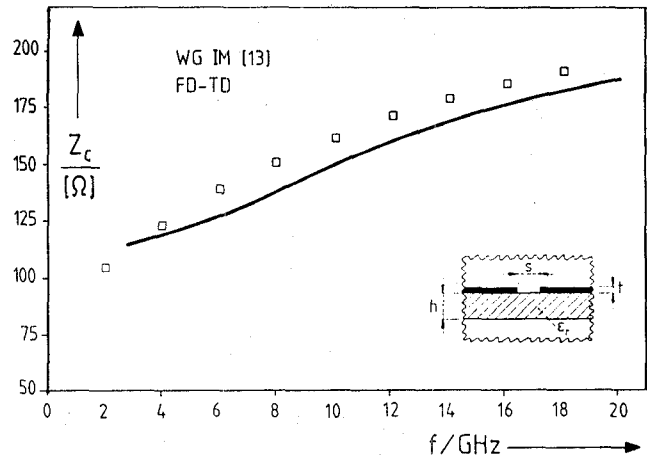
Characteristic impedance (power-current definition) for the open microstrip line.  $\epsilon_r = 9.7$ ,  $t = 0$ . Discretization:  $35 \times 25$ .

For the slot-line (Fig. 6) and the coplanar line (Fig. 7) again the power-voltage definition of the characteristic impedance is applied. In the case of the coplanar line, this definition approximates best the quasi-static results reported in [14], [15], as compared with the other possible definitions (voltage-current, power-current), cf. Fig. 7.



**Fig. 5:**

Characteristic impedance (power-current definition) for a lateral open triplate structure.  $\epsilon_r = 4$ . Discretization:  $30 \times 10$ .



**Fig. 6:**

Characteristic impedance (power-voltage definition) for an open slotline.  $\epsilon_r = 9.7$ ,  $s = 1.27\text{mm}$ ,  $h = 0.635\text{mm}$ ,  $t = 0$ .

Discretization:  $30 \times 50$ .

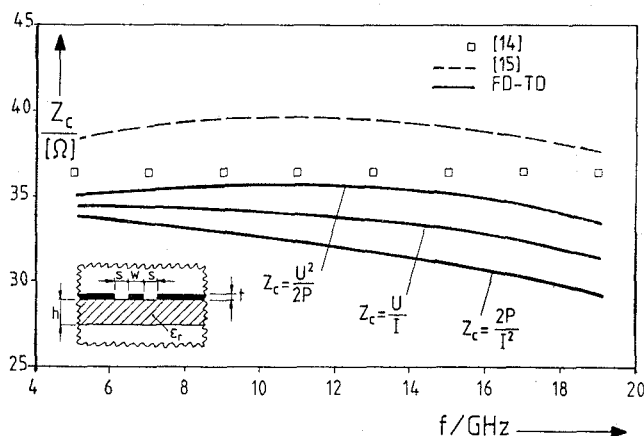


Fig. 7:

Characteristic impedance (power-voltage definition, together with the power-current and voltage-current definitions) for an open coplanar line.  $\epsilon_r = 3.7$ ,  $t = 0$ ,  $d = 0.127\text{mm}$ ,  $w = s = 0.086\text{mm}$ . Discretization:  $28 \times 20$ .

## CONCLUSION

A simplified 2D graded mesh FD-TD algorithm is described for the full-wave calculation of the characteristic impedance of a class of open or shielded planar waveguiding structures including the finite metallization thickness. Since the numerical discretization is restricted only to the cross-section of the waveguide, the efficiency and the flexibility of this algorithm makes the FD-TD method an attractive CAD tool for microwave and mm-wave applications. The theory is verified by very good agreement with results obtained by classical methods.

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